Note: You are only required to attempt ONLY ONE of following questions.

(Q14a) End Game

A matrix is a rectangular array or a table of numbers, symbols or expressions, arranged in rows or columns. Here, we are concerned about 2×2 matrices, i.e. matrices with two rows and two columns. If

$$A := \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B := \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

are two matrices where a, b, c, d, e, f, g, h are complex numbers (search the internet if you don't know what complex numbers are), then we add A and B in the following manner.

$$A + B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}.$$

We add component-wise. We multiply a matrix with a number m, which can be complex as well, as follows:

$$mA = m \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ma & mb \\ mc & md \end{bmatrix}.$$

We multiply two matrices in the following *strange* manner.

$$A \cdot B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}.$$

If you're seeing matrix multiplication for the first time, don't get worried. You shall see why we've defined matrix multiplication in such peculiar manner in your future studies. We define a special matrix called the identity matrix, denoted by $\mathbb{I}$, as follows:

$$\mathbb{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

With this much information, can you come up **three** matrices M_1, M_2 and M_3 such that

(i) $M_1 \cdot M_1 = M_2 \cdot M_2 = M_3 \cdot M_3 = \mathbb{I}$;

(ii) $M_1 \cdot M_2 = -M_2 \cdot M_1 = iM_3$, where $i = \sqrt{-1}$ (yes, such things exist! lookup complex numbers);

(iii) $M_1 \cdot M_2 \cdot M_3 = i\mathbb{I}$ (note that matrix multiplication is associative, but not commutative, in general);

(iv) the determinant of M_1, M_2 and M_3 equals to -1;

(v) the sum of the diagonal elements of each of the matrices vanishes. As an example, for the matrix A that we've defined, this condition equals saying that $a + d = 0$.

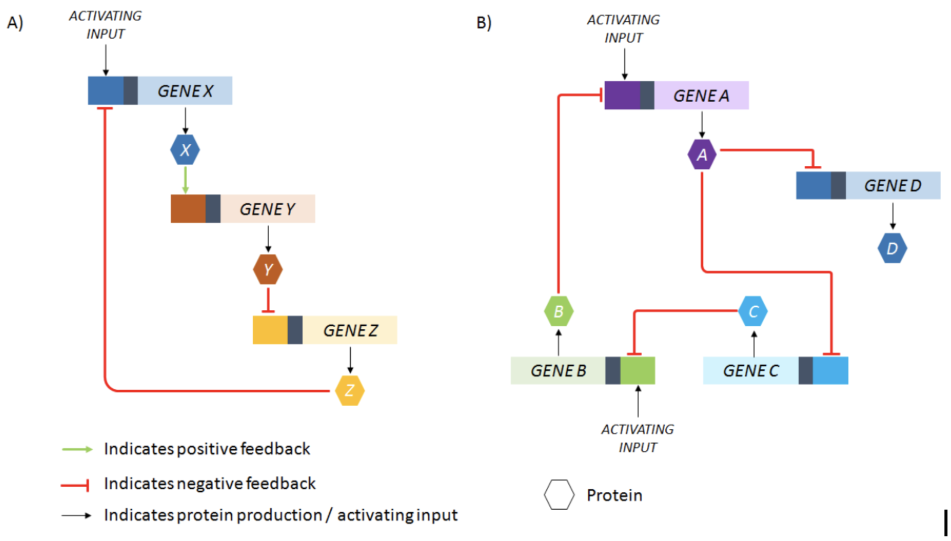
The **product of the diagonal elements** of the matrix $M_1 + M_2 + M_3$ is the magic number you desire.

(Q14b)

This question is a contribution from Team iGEM, IISER Bhopal.

Prof. Robert has recently identified the presence of feedback loops in a newly discovered bacteria Hypothetica verificata. Feedback loops are biological mechanisms that allow organisms to maintain homeostasis. They consist of genes (sequences of DNA) which give rise to proteins. These genes can interact with each other. The protein from one gene involved in the loop leads to either an increase (positive feedback loop) or decrease (negative feedback loop) in the production of itself or another gene. These processes can be direct or indirect and are of great consequence to the functioning of an organism.

Given below are representations of the feedback loops Prof. Robert has identified in H. verificata. Try to qualitatively predict the behaviour of these genetic circuits (explain each).



Note that positive feedback refers to activation, while negative feedback refers to inhibition.

For A) Assume that no protein (X, Y, Z) is present in the system and only upon receiving the "Activating Input" do the genes start producing proteins.

For B) Assume that only Proteins A C are already present the system and Protein B is absent.

(Hint: This means that only Gene A will continue producing Protein A. Also, Protein C will eventually be degraded by H. verificata since it is not being produced)